ON THE THEORY OF PERFORMANCE OF WIDE AND ULTRA-WIDE BAND LATTICE TYPE CRYSTAL BAND-PASS FILTERS CONTAINING STABILISED NEGATIVE IMPEDANCE ELEMENTS

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ABSTRACT. The paper relates to the theory of performance of wide and ultra-wide band lattice type crystal band-pass filter sections containing crystal, capacitance, inductance and stabilized negative impedance elements developed by the author elsewhere. One typical section of each of th eClasses I and II has been chosen for consideration.

The typical lattice section of Class I consists of a series resonant element in the series arm and a crystal (mounted between two electrodes) connected in series with a stabilised negative impedance element in the lattice arm. Five important cases have been considered. Nature of reactances in series and lattice arms for the five cases with their subcases has been investigated, and the possibility of attenuation peak and '' or waviness in total insertion-loss characteristics has been discussed. The lattice section has been reduced to an equivalent T-section for calculation purpose. Characteristic impedance and its nature have been discussed. The cut-off frequencies have been determined and the attenuation and phase constants of the section have been obtained.

The typical lattice section of Class II consists of a crystal (mounted between two electrodes) connected in parallel to a stabilised negative impedance element in the series arm and a crystal (mounted between two electrodes) connected in series with a stabilised negative impedance element in the lattice arm. Four important cases have been considered. Nature of reactances in series and lattice arms for the four cases with their subcases has been investigated. The lattice section has been reduced as before to an equivalent T-section. Characteristic impedance and its nature have been discussed. The cut-off frequencies have been determined and the attenuation and phase constants of the section have been obtained.

I. INTRODUCTION

The simplest type of crystal band-pass filter is the ladder section (Mason, 1934) consisting of crystal and capacitance elements which gives a small transmission band-width. By using a lattice section (Mason, 1934) employing crystal and capacitance elements, it is possible to get a larger band-width which is still much less than the requirement. Lattice section (Mason, 1934, 1937; Stanesby and Broad, 1939, 1941; Stanesby, 1942) using crystal, capacitance and inductance elements is capable of giving much larger band-width than the lattice section without the inductance element, but this still remains much narrower for majority of requirements of the modern communication systems. Besides, the use of inductance coils of comparatively lower 'Q' value gives large attenuation in the transmission band.

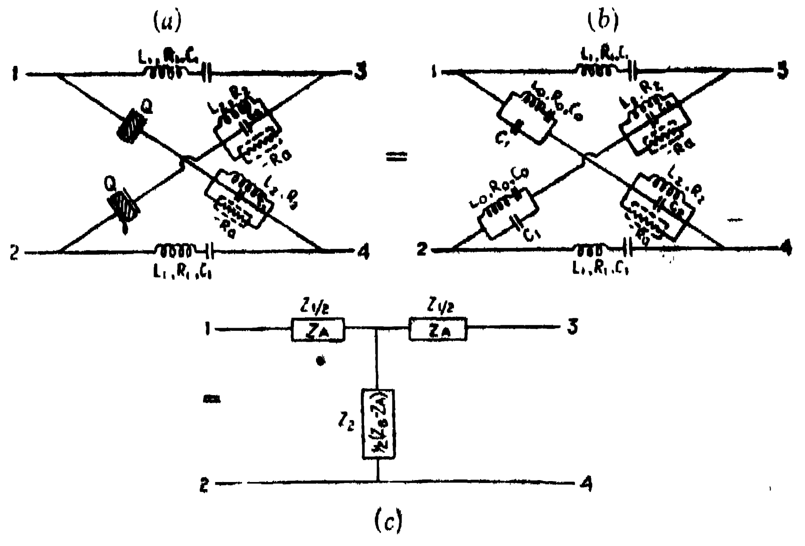
The author in a previous paper (Chakravarti and Dutt, 1940) developed wide band and ultra-wide band low-loss lattice type crystal band-pass filters containing crystal, capacitance, inductance and stabilised negative impedance elements. The characteristic impedance and total insertion loss characteristics of such lattice sections have been measured and the sharpness of cut-off on the two sides has been compared to that of lattice sections using crystal, capacitance and inductance elements.

The wide and ultra-wide band crystal band-pass filters so developed have considerable applications in several types of modern communication systems—for example, as band-pass filters in television and broad band carrier current systems and as band-pass filters and band-pass couplings in multi-channel radio telephone transmission and reception systems. By reducing the band-width to less or somewhat less than that required for the above communication systems (which is possible by the methods developed), they are capable of being utilized for various other useful purposes in short-wave and medium wave radio systems.

The present paper relates to further investigation regarding the performance of wide and ultra-wide band lattice type crystal band-pass filter sections (designated as Class I and Class II types in the author's previous paper) using crystal, capacitance, inductance and stabilized negative impedance elements.

II. WIDE AND ULTRA-WIDE BAND LATTICE TYPE CRYSTAL BAND-PASS FILTER SECTION—CLASS I

Consider a typical Class I lattice section in which a series resonant element (consisting of an inductance and a capacitance connected in series) is in the series arm and a crystal (mounted between two electrodes) connected in series with a stabilized negative impedance element is in the lattice arm [Fig. 1(a)]



(c)

FIG. 1

1. DIFFERENT CASES

Five important cases may arise as follows—(1) in which the negative impedance element is detuned to the crystal frequency as well as to the resonance frequency of the series resonant element; (2) in which the negative impedance element is tuned to the resonance frequency of the series resonant element but the crystal frequency is different; (3) in which the negative impedance element is tuned to the crystal frequency but the resonance frequency of the series resonant element is different; (4) in which the series resonant element is tuned to the crystal frequency but the negative impedance element is tuned to a different frequency; (5) in which the negative impedance element is tuned to the crystal frequency as well as the resonance frequency of series resonant element.

Suppose F_1 = resonance frequency of the series resonant element (in the series arm), F_2 = resonance frequency of the parallel resonant element incorporated in the stabilized negative impedance element (in the lattice arm), f_0 = resonance frequency of the crystal (in the lattice arm), and F_0 = overall frequency (or anti-resonance frequency) of the crystal (in the lattice arm). Then in case (1) F_1, F_2 and F_0 (or f_0) are all different; in case (2) $F_1 = F_2$ but F_0 (or f_0) is different; in case (3) $F_2 = F_0$ (or f_0) and F_1 is different; in case (4) $F_1 = F_0$ (or f_0) and F_2 is different; and in case (5) $F_1 = F_2 = F_0$ (or f_0)

2. NATURE OF REACTANCES IN SERIES AND LATTICE ARMS

Fig. 2 shows the reactance-frequency characteristics of the series and lattice arms drawn for the five cases mentioned above. The curve marked I refers to the characteristic of the series resonant element in the series arm; the curve marked II_1 refers to that of the negative impedance element in the lattice arm, and the curve marked II_2 refers to that of the crystal (mounted between electrodes) in the lattice arm.

The nature of the reactance characteristic of the negative impedance element in the lattice arm follows from the following calculations. The impedance of the negative impedance element is given by

$$_{N}Z_2 = \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2} \qquad \ldots \quad (1)$$

The reactance component of $_{N}Z_2$ can be shown to be

$$j_N X_2 = j.\frac{\omega L_2 R_a^2(1-\omega^2 L_2 C_2)}{R_a^2 R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2} \qquad \ldots \quad (2)$$

$$= -j.\frac{L_2 C_2\left(\omega L_2 - \frac{1}{\omega C_2}\right)}{C_2^2\left(\omega L_2 - \frac{1}{\omega C_2}\right)^2 + \frac{L_2^2}{R^2}}, \qquad \ldots \quad (2a)$$

on dividing both numerator and denominator by $\omega_2 R_a^2$.

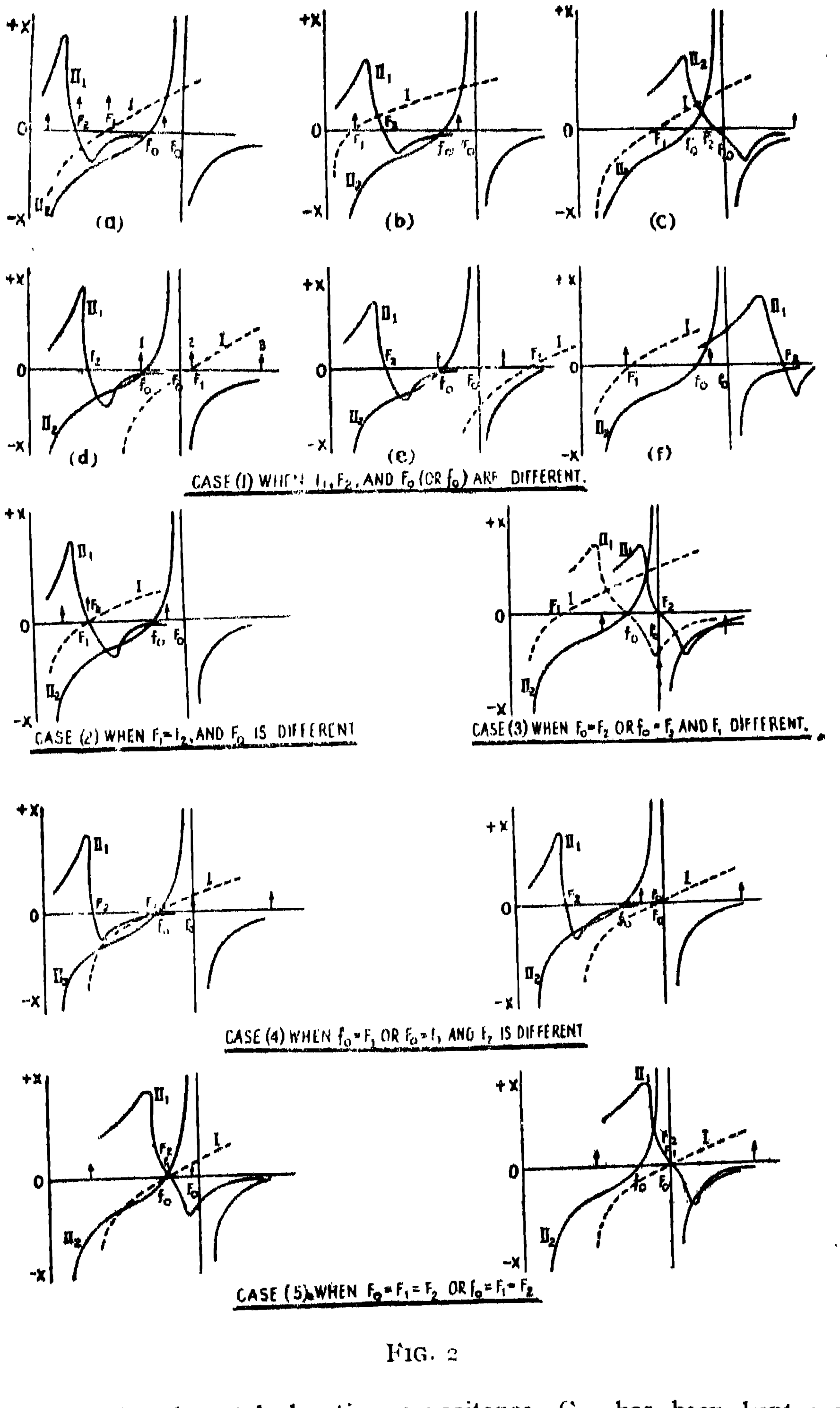


FIG. 2

•Assume that the total shunting capacitance C_2 has been kept more or less constant at a suitable value (*i.e.*, by means of an external capacitance) when h.f. voltage of varying frequency is impressed on the negative impedance element ; that the resonance frequency of L_2-C_2 parallel combination associated with the negative impedance element is 'F_2' Mc/s. Therefore with h.f. voltage of varying frequency applied to the negative impedance element, the magnitude of the resistance component *only* has to be taken to change with ω according to the approximate relation $/R_n/ = \frac{m}{\omega^2}$ [found by the author elsewhere (Chakravarti and Das, 1953)] where m is a constant. Then

$$_NX_2 = -\frac{L_2C_2\left(\omega L_2 - \frac{1}{\omega C_2}\right)}{C_2^2\left(\omega L_2 - \frac{1}{\omega C_2}\right)^2 + \frac{L_2^2}{m^2}\omega^4} \quad \ldots \quad (3)$$

When $\omega < \frac{1}{\sqrt{L_2C_2}}$, $\omega L_2 - \frac{1}{\omega C_2}$ will be negative and hence $_NX_2$ will be positive;

when $\omega = \frac{1}{\sqrt{L_2C_2}} = 2\pi F_2$, $_NX_2 = 0$; and when $\omega > \frac{1}{\sqrt{L_2C_2}}$, $\omega L_2 - \frac{1}{\omega C_2}$ will be positive and hence $_NX_2$ will be negative.

Starting from a point before F_2 as the frequency is increased the magnitude of the positive reactance has been found to increase till a frequency say $F_2 - f'$ and then to decrease rapidly to zero value at F_2. After the resonance frequency F_2 is passed, the reactance is of negative sign and its magnitude at first increases till a certain frequency say $F_2 + f''$ and then decreases to low value as the frequency is further increased. The form of the reactance curve is shown in Fig. 3.

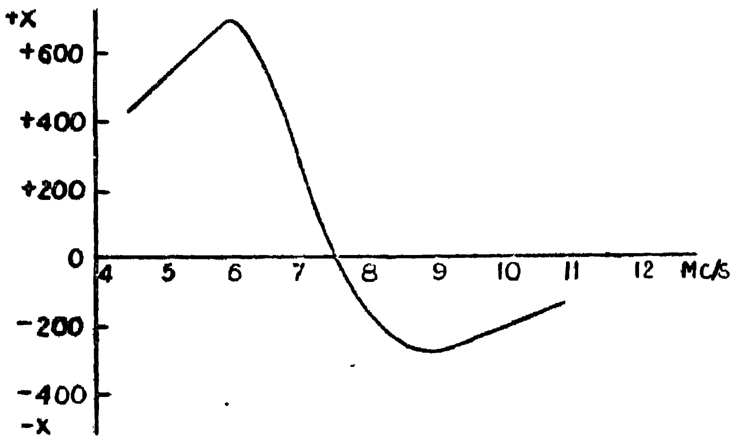


Fig. 3

The nature of the reactance characteristic of a typical negative impedance element can be seen from figures in Table I. $L_2 = 10 \times 10^{-6}$H; $C_2 = 44 \times 10^{-12}$F; $R_a = 2 \times 10^3$ ohms at 5 Mc/s, 1.4×10^3 ohms at 6 Mc/s, 0.9×10^3 ohms at 7.6 Mc/s, 0.8×10^3 ohms at 8 Mc/s and 0.68×10^3 ohms at 9 Mc/s, 0.5×10^3 ohms at 10 Mc/s and $F_2 = 7.6$ Mc/s roughly.

TABLE I

Frequency Mc/s	L_2C_2	$\omega L_2 - \frac{1}{\omega C_2}$	$C_2^2\left(\omega L_2 - \frac{1}{\omega C_2}\right)^2 + \frac{L_2^2}{R_a^2}$	$_NX_2$ ohms
5	440×10^{-18}	-406	3.25×10^{-10}	$+540$
6	440×10^{-18}	-223	1.40×10^{-10}	$+691$
7.6	440×10^{-18}	0	1.23×10^{-10}	0
8	440×10^{-18}	$+54$	1.76×10^{-10}	-135
9	440×10^{-18}	$+167$	2.63×10^{-10}	-279
10	440×10^{-18}	$+270$	5.6×10^{-10}	-213

It will be noted that such a variation of the reactance of negative impedance element as shown above added on to the typical variation of the reactance of a crystal agrees well with the variation of the measured value of total reactance 'X_2' in the lattice arm with frequency for the untuned case given in Fig. 8, on page 305 of the author's previous paper (Chakravarti and Dutt, 1940).

It will be seen from the reactance diagrams drawn for different cases in Fig. 2 that the performance of wide band and ultra-wide band band-pass filters depends not only upon the values of F_1, F_2 and F_0 (or f_0) but upon their location in the frequency spectrum with respect to one another.

Further, in all cases and sub cases the probable cut-off frequencies indicated on the diagrams are on the basis that the impedances in series and lattice arms are purely reactive. In actual practice the impedances have resistance components as well and the results get modified. For example, if the series resonant element in the series arm be taken to contain no resistance, it is evident that at F_1 the reactance is zero and Z_A (where Z_A is the total impedance in each series arm of the lattice section) is also zero. Hence the characteristic impedance of the lattice section becomes zero giving a cut-off frequency at F_1. On the other hand, if the series resonant element in the series arm has a resistance component, the characteristic impedance of the section at F_1 is not zero; and the total insertion loss characteristic will be characterised by reflection effects giving the 'effective cut-off frequency' somewhat different from F_1.

Take case (1) in which F_1, F_2 and F_0 (or f_0) are different. (*a*) shows the condition when $F_2 < F_1 < F_0$ (or f_0), and F_2 and F_1 are nearer to each other. It will be observed that the reactance in the series arm and the total reactance in the lattice arm are of the same sign between F_2 and F_1 with the magnitude of the latter reactance greater than that of the former and further the reactance in the series arm becomes zero at F_1. Consequently it can be expected that there will be large attenuation and/or waviness in the total insertion loss characteristic between F_2 and F_1. This condition may give rise to two band-pass filters having almost adjoining transmission bands or a single band-pass filter of larger band-width with waviness about F_1 according as the resistance component of the impedance in the series arm is negligible or has appreciable value and also according as F_2 is farther away or much nearer to F_1;

(*b*) shows the condition when $F_1 < F_2 < F_0$ (or f_0), and F_1 and F_2 are nearer to each other. The probable positions of cut-off frequencies are indicated by arrows. In this case the transmission band-width will be large and the cut-off will be equally sharp on both the sides. The total insertion loss characteristic over the transmission band is expected to be fairly even;

(*c*) shows the condition when $F_1 < f_0 < F_2 < F_0$, and F_1 and F_2 are nearer to f_0. The probable positions of cut-off frequencies are indicated by arrows. In this case the transmission band-width will be large and the cut-off will not be equally sharp on both the sides;

(*d*) shows the condition when $F_2 < F_0$ (or f_0) $< F_1$, and F_1 is nearer to F_0. The transmission band-width may be small as indicated by arrows 1-2, or may be large as indicated by arrows 1-3 with waviness in the total insertion loss characteristic about F_1. The sharpness of cut-off is expected to be unequal on the two sides;

(*e*) shows the condition when $F_2 < F_0$ (or f_0) $< F_1$, and F_1 is far away from F_0. This condition is expected to give small transmission band-width and the sharpness of cut-off may be more or less the same on both the sides;

(*f*) shows the condition when $F_1 < F_0$ (or f_0) $< F_2$, and F_2 is far away from F_0. This condition may be suitable for wide and ultra-wide band band-pass filters. The nearer F_2 is to F_0, the larger will be the transmission band-width. The sharpness of cut-off is expected to be the same on the two sides.

Take case (2) in which $F_1 = F_2$, and F_0 or f_0 is different. This case refers to F_1 (or F_2) being less than F_0, but a similar diagram could be made for F_1 (or F_2) being greater than F_0. It will be seen that at F_1 (or F_2) the reactance of series arm becomes zero and the characteristic impedance will become zero unless modified by the resistance component of the series arm impedance. This condition may give rise to two band-pass filters having adjoining transmission bands or a single band-pass filter of larger band-width with waviness in the total insertion loss characteristic about F_1 (or F_2) for reasons given in (*a*) of case (1).

Take case (3) in which $F_0 = F_2$, or $f_0 = F_2$, and F_1 is different and less than F_2. It will be seen that when $f_0 = F_2$ the band-width is indicated by arrows (placed on the frequency axis). When $F_0 = F_2$ the band-width is smaller and indicated by arrows (placed along the frequency axis at a lower level). The sharpness of cut-off is expected to be different on the two sides.

Take case (4) in which $F_0 = F_1$ or $f_0 = F_1$, and F_2 is different and less than F_1. It will be seen that when $F_0 = F_1$, or $f_0 = F_1$ the expected band-width may be more or less the same and is indicated by the arrows on the two diagrams. The sharpness of cut-off is expected to be different on the two sides.

Take case (5) in which $F_0 = F_1 = F_2$, or $f_0 = F_1 = F_2$. It will be seen that when $f_0 = F_1 = F_2$, the band-width (marked by arrows) is slightly smaller than when $F_0 = F_1 = F_2$. The sharpness of cut-off is expected to be similar on both the sides.

3. ON POSSIBLE ATTENUATION PEAK AND WAVINESS IN TOTAL INSERTION-LOSS CHARACTERISTICS IN CERTAIN CASES

If the reactances in series and lattice arms are of opposite signs and same in magnitude over the transmission band, the characteristic impedance in the transmission band is a pure resistance of magnitude same as that of one of the reactances. If Z_A and Z_B are the total impedances in each series and each lattice arms respectively, suppose $Z_A = +jX$ and $Z_B = -jX$. Then $Z_0 = \sqrt{Z_A Z_B} = \sqrt{X^2}$, so that the magnitude of characteristic impedance is $/X/$. When terminated by non-reactive impedance 'X', the reflection loss in the transmission band is zero and the total insertion loss or gain will be due to network loss or gain respectively.

If the reactances in series and lattice arms are of opposite signs but very much unequal in magnitudes, the characteristic impedance will be a pure resistance of value smaller or larger than $/X/$. When still terminated by non-reactive impedance 'X,' there will be reflection loss (depending upon the magnitudes of the impedances) to be added on to the network attenuation or gain.

If one of the reactances say in series arm is zero and the other one in lattice arm either of positive or of negative sign, the characteristic impedance is zero. When terminated by non-reactive impedance 'X,' there will be very large reflection loss giving a kind of attenuation peak in the total insertion-loss characteristic. This may be modified by the resistance component of the series arm impedance (if appreciable) since in that case the characteristic impedance will not be zero but will contain both resistance and reactance components.

If both the reactances are of the same sign and their magnitudes are nearly equal or unequal, the characteristic impedance is purely reactive. When terminated by a non-reactive impedance 'X,' there will be large reflection loss with reflection phase shift giving large attenuation as well as waviness in the total insertion loss characteristic. The condition discussed here is typical for the attenuation band of a band-pass filter.

The waviness in the total insertion loss characteristic due to reflection effects could be noticed in the curve for I(1)D type band-pass filter shown in Fig. 7 (page 304) of the author's previous paper (Chakravarti and Dutt, 1940).

4. EQUIVALENT T-SECTION OF THE ORIGINAL LATTICE SECTION

A general section (in which the negative impedance element is detuned to the crystal frequency as well as to the resonance frequency of series arm) is shown in Fig. 1(*a*) and its effective equivalent is shown in Fig. 1(*b*). Neglect R_1, R_0 and R_2, the resistance components in the various inductances.

If Z_A be the total impedance in *each* series arm and Z_B the total impedance in *each* lattice arm, then we have

$$Z_A = j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \qquad \dots \quad (4)$$

$$Z_B = \frac{-\frac{j}{\omega C_0}(1-\omega^2 L_0 C_0)}{1-\omega^2 L_0 C' + \frac{C'}{C_0}} + \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2}. \quad \ldots \quad (5)$$

For a crystal mounted between two electrodes,

$$\frac{C'}{C_0} = k \quad \ldots \quad (6)$$

where 'k' is about 140 for the type of quartz crystal used by the author. The expression for Z_B can be simplified by putting $C_0 k$ for C' from (6) as follows :—

$$Z_B = \frac{-\frac{j}{\omega C_0}(1-\omega^2 L_0 C_0)}{1 + k - k\omega^2 L_0 C_0} + \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2}. \quad \ldots \quad (7)$$

Since '1' can be neglected in comparison to 'k' in the denominator of the first term on the R.H.S. of equation (7) without appreciable error

$$Z_B \simeq -\frac{j}{\omega C_0 k} + \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2}. \quad \ldots \quad (8)$$

Now since the lattice section in Fig. 1(b) is equivalent to the T-section in Fig. 1(c), the various results for the lattice section can be obtained from those of the equivalent T-section.

If Z_1 and Z_2 be the total series and total shunt impedances of the equivalent T-section respectively in Fig. 1(c), then (deducing from Figs. 24A and 24B, Appendix 'D,' page 281, of Transmission Circuits for Telephonic Communication by K. S. Johnson) we have

$$Z_1 = 2Z_A = 2j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \quad \ldots \quad (9)$$

$$Z_2 = \tfrac{1}{2}(Z_B - Z_A) = \tfrac{1}{2}\left[\frac{\omega^2 L_2^2 R_a}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2} - j\left\{\frac{1}{\omega C_0 k} - \frac{\omega L_2 R_a^2(1-\omega^2 L_2 C_2)}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2} + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\right\}\right] \quad \ldots \quad (10)$$

5. CHARACTERISTIC IMPEDANCE

The characteristic impedance 'Z_0' of the T-section is given by

$$Z_0 = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

$$= \sqrt{\left(\omega L_1 - \frac{1}{\omega C_1}\right)\left[\frac{1}{\omega C_0 k} - \frac{\omega L_2 R_a^2(1-\omega^2 L_2 C_2)}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2} - \frac{j.\omega^2 L_2^2 R_a}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2}\right]} \quad \ldots \quad (11)$$

The imaginary term under the radical sign $\left|\frac{\omega^2 L_2^2 R_a}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2}\right|$ is a small fraction of the first real term $\left|\frac{1}{\omega C_0 k}\right|$ over a large frequency band, as it can be shown by the example given below. The second real term is much smaller than the first one. As ω increases, the first real term becomes smaller and smaller with the result that after a certain frequency limit the imaginary term becomes quite appreciable compared to it.

Take a typical wide band lattice section of class I having a comparatively smaller band-width so that $|R_a|$ could roughly be taken to vary from 2×10^3 ohms to $0{\cdot}8\times10^3$ ohms over the band under consideration. For this section suppose $F_1=5.0$ Mc/s, $f_0=7.17$ Mc/s, $F_0=7{\cdot}24$ Mc/s, $F_2=7.6$ Mc/s, $L_1=10\times10^{-6}$H, $C_1=101.6\times10^{-12}$F, $C_0=.011\times10^{-12}$F, $L_2=10\times10^{-6}$H, $C_2=44\times10^{-12}$F, $|R_a|=2\times10^3$ ohms at 5 Mc/s, 1.4×10^3 ohms at 6 Mc/s, 1.0×10^3 ohms at 7.3 Mc/s and 0.8×10^3 at 9 Mc/s and $k=140$.

Then the ratio of the first real term to the imaginary term under radical sign in (11) is given by

$$N=\frac{1}{\omega C_0 k}\Bigg/\frac{\omega^2 L_2^2 R_a}{R_a^2(1-\omega^2L_2C_2)^2+\omega^2L_2^2}=\frac{R_a^2(1-\omega^2L_2C_2)^2+\omega^2L_2^2}{\omega^3 C_0 L_2^2 R_a k}\ \ldots \quad (12)$$

It will be seen that $N=135$ at 5 Mc/s, $N=34$ at 6 Mc/s, $N=17$ at 7 Mc/s, $N=16$ at 7.3 Mc/s., and $N=16$ at 9 Mc/s.

Hence the imaginary term may be dropped out in comparison to the real terms under the radical sign at least over the frequency range 5.0-9.0 Mc/s (the lower limit has been chosen from the consideration that at 5 Mc/s, Z_0 will be zero). Similar calculations can be obtained for almost all other sections of this class.

$$\therefore\quad Z_0\sim\sqrt{\left(\omega L_1-\frac{1}{\omega C_1}\right)\left[\frac{1}{\omega C_0 k}-\frac{\omega L_2 R_a^2(1-\omega^2L_2C_2)}{R_a^2(1-\omega^2L_2C_2)^2+\omega^2L_2^2}\right]}\ \ldots \quad (13)$$

which is purely non-reactive and depends upon ω.

6. NATURE OF THE CHARACTERISTIC IMPEDANCE

$\omega L_1-\frac{1}{\omega C_1}$ will be negative for values of $\omega<\frac{1}{\sqrt{L_1C_1}}$; it will be zero for $\omega=\frac{1}{\sqrt{L_1C_1}}=2\pi F_1$ and will be positive for $\omega>\frac{1}{\sqrt{L_1C_1}}$.

Therefore Z_0 will be pure reactance for frequencies less than F_1, it will be zero at F_1 and will be almost pure resistance between F_1 and a frequency f_2 since the reactance component will be negligibly small between these limits. At frequencies greater than f_2, the reactance component of Z_0 becomes appreciably large compared to the resistance component and hence Z_0 becomes an impedance of more and more reactive nature.

The characteristic impedance of the wide band section mentioned above over the range 5-9 Mc/s is shown in Table II.

TABLE II

f in Mc/s ...	5	6	7	7·3	9
Z_0 in ohms ...	0	1386	1794	1852	2100

It will be seen that Z_0 increases with 'f' (at first increasing rapidly and later on slowly) over the range 5-9 Mc/s which is the transmission band of the section as shown in the following sections.

7. CUT-OFF FREQUENCIES

By referring to the section on the "Nature of reactances in series and lattice arms," it may be predicted that one of the cut-off frequencies will lie at or near about F_1 and another of the cut-off frequencies near about the other edge of the band over which Z_0 is more or less non-reactive.

Since Z_0 is expected to be non-reactive over the transmission band, both Z_1 and Z_2 are to be purely reactive and of opposite signs over the same band. From (9), Z_1 is purely reactive. From equation (10), Z_2 has both resistance and reactance components ; but since

$$\left| \frac{\omega^2 L_2^2 R_a}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2} \right|$$

has been neglected in comparison to

$$\left| \frac{1}{\omega C_0 k} \right|$$

in section 5 on 'characteristic impedance,' it should be similarly done *as well* in equation (10) to obtain Z_2 as purely reactive.

Therefore, for calculation of the cut-off frequencies (*i.e.* the limits between which Z_0 is more or less non-reactive),

$$Z_1 = 2j\left(\omega L_1 - \frac{1}{\omega C_1}\right), \qquad \text{[same as (9)]}$$

$$\text{and } Z_2 \simeq -\frac{j}{2}\left[\frac{1}{\omega C_0 k} - \frac{\omega L_2 R_a^2(1-\omega^2 L_2 C_2)}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2} + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\right]. \quad \cdots \quad (14)$$

$$\text{If } \frac{Z_1}{Z_2} = 0, \text{ then } \omega^2 = \frac{1}{L_1 C_1}, \text{ or } f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}} = F_1. \quad \cdots \quad (15)$$

Hence lower cut-off frequency $f_1 = 5{\cdot}0$ Mc/s for the section taken.

$$\text{If } \frac{Z_1}{Z_2} = -4, \text{ then } \frac{1}{\omega C_0 k} = \frac{\omega L_2 R_a^2(1-\omega^2 L_2 C_2)}{R_a^2(1-\omega^2 L_2 C_2)^2 + \omega^2 L_2^2}, \text{ ultimately}$$

or $$\omega^4(C_2 + C_0 k)L_2^2 C_2 R_a^2 - \omega^2(2R_a^2 C_2 - L_2 + C_0 k R_a^2)L_2 + R_a^2 = 0.$$

Solving the biquadratic equation and putting $2\pi f_2$ for ω, we have

$$f_2 = \frac{1}{2\pi}\left[\frac{(2R_a^2C_2 + R_a^2C_0k - L_2) \pm \{(2R_a^2C_2 + R_a^2C_0k - L_2)^2 - 4R_a^4C_2(C_2 + C_0k)\}^{\frac{1}{2}}}{2L_2C_2R_a^2(C_2 + C_0k)}\right]^{\frac{1}{2}} \quad \ldots \quad (16)$$

where 'f_2' is the real admissible value.

Taking values of C_0, L_2, C_2 and k and appropriate value of $|R_a|$ at the frequency concerned, $\omega = \sqrt{3226 \times 10^{12}} = 56.8 \times 10^6$ *r.p.s.*,

$$\therefore \qquad f_2 = 9.06 \text{ Mc/s.}$$

Hence the cut-off frequencies of the typical section considered are 5.0 and 9.06 Mc/s, or F_1 and f_2 Mc/s. It may be said that if F_0 be the overall frequency of the crystal in the lattice arm, the cut-off frequencies of the section are *very roughly* $F_0 - \frac{B}{2}$ and $F_0 + \frac{B}{2}$ where $\frac{B}{2} \simeq 2{,}03$ Mc/s.

8. ATTENUATION AND PHASE CONSTANTS

The ratio of the current at the input (I_1) to the current at the output (I_2) when the equivalent T-section is terminated in Z_0 in the transmission band is given by

$$\frac{I_1}{I_2} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

$$= 1 - \frac{2\left(\omega L_1 - \frac{1}{\omega C_1}\right)}{\left[\frac{1}{\omega C_0 k} - \frac{\omega L_2 R_a^2(1 - \omega^2 L_2 C_2)}{R_a^2(1 - \omega^2 L_2 C_2)^2 + \omega^2 L_2^2} + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\right]}$$

$$+ j \cdot \frac{2Z_0}{\left[\frac{1}{\omega C_0 k} - \frac{\omega L_2 R_a^2(1 - \omega^2 L_2 C_2)}{R_a^2(1 - \omega^2 L_2 C_2)^2 + \omega^2 L_2^2} + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\right]} \quad \ldots \quad (17)$$

Let $l = \frac{1}{\omega C_0 k} - \frac{\omega L_2 R_a^2(1 - \omega^2 L_2 C_2)}{R_a^2(1 - \omega^2 L_2 C_2)^2 + \omega^2 L_2^2}$ and $m = \omega L_1 - \frac{1}{\omega C_1}$.

Then from (13), $\qquad Z_0 \simeq \sqrt{lm}$.

The equation (17) can be written as

$$\frac{I_1}{I_2} = 1 - \frac{2m}{l+m} + j \cdot \frac{2\sqrt{lm}}{l+m} = \frac{l-m}{l+m} + j \cdot \frac{2\sqrt{lm}}{l+m}. \quad \ldots \quad (18)$$

If P = propagation constant = $\log_e(I_1/I_2) = \alpha + j\beta$ where α = attenuation constant and β = phase constant of the section, then

$$\alpha = \log_e \sqrt{\left(\frac{l-m}{l+m}\right)^2 + \frac{4lm}{(l+m)^2}} = \log_e \sqrt{\frac{(l-m)^2 + 4lm}{(l+m)^2}} = \log_e 1 = 0. \quad (19)$$

Hence there is no attenuation in the transmission band, F_1 to f_2 Mc/s. Actual measurements of network attenuation have shown *low loss* in the transmission band for the sections of this class. Further,

$$\beta = \tan^{-1} \frac{2\sqrt{lm}}{l-m}. \qquad \ldots \quad (20)$$

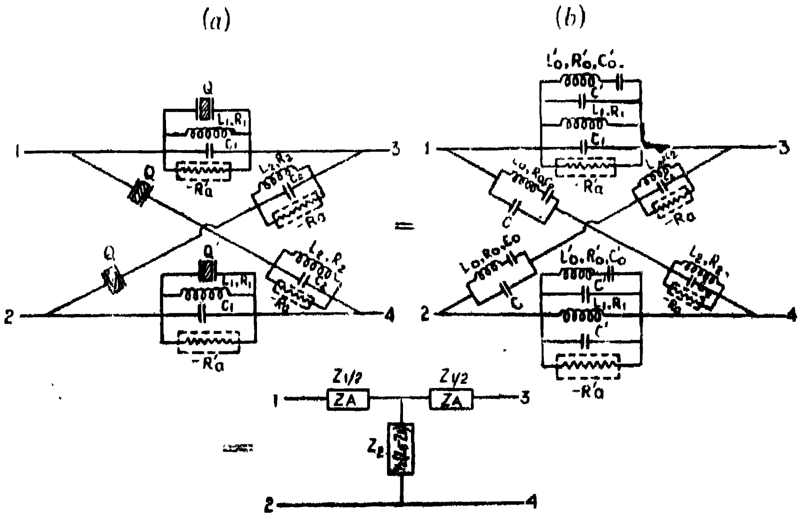
Table III shows the values of β over the frequency range 5-9 Mc/s for the lattice section considered.

TABLE III

f in Mc/s ...	5	6	7	7.22	9
β in degrees...	0	9° 18′	14° 6′	14° 48′	21° 24′

III. WIDE AND ULTRA-WIDE BAND LATTICE TYPE CRYSTAL BAND-PASS FILTER SECTION—CLASS II

Consider a typical class II ultra-wide band section in which a crystal (mounted between electrodes) connected in parallel to a stabilised negative impedance element is in the series arm and a crystal (mounted between two electrodes) connected in series with a stabilised negative impedance element is in the lattice arm [Fig. 4(*a*)].



(*c*)
FIG. 4

1. DIFFERENT CASES

Four important cases may arise as follows—(1) in which the resonance frequency of the *equivalent* parallel resonant circuit formed from $L_1 - C_1$ combination associated with the stabilised negative impedance element and equivalent circuit of the crystal (mounted between two electrodes) in the series arm, the frequency of the crystal in the lattice arm and the resonance frequency of the parallel resonant circuit in the stabilised negative impedance element in the lattice arm are all different; (2) in which the resonance frequency of the *equivalent* parallel resonant circuit referred to above in the series arm is equal to the resonance frequency of the parallel resonant circuit incorporated in negative impedance element in the lattice arm, and the frequency of crystal in the lattice arm is different; (3) in which the resonance frequency of the *equivalent* parallel resonant circuit (referred to above) in the series arm is equal to the frequency of the crystal (resonance or anti-resonance frequency) in the lattice arm, and the resonance frequency of the parallel resonant circuit incorporated in negative impedance element in the lattice arm is different; and (4) in which the resonance frequency of the *equivalent* parallel resonant circuit (referred to above) in the series arm, the resonance frequency of the parallel resonant circuit incorporated in negative impedance element in the lattice arm and the frequency of the crystal in the lattice arm are all the same.

Suppose F_1 = resonance frequency of the *equivalent* parallel resonant circuit (referred to above) in the series arm, F_2 = resonance frequency of the parallel resonant circuit in the stabilised negative impedance element in the lattice arm, f_0 = resonance frequency of the crystal in the lattice arm and F_0 = over all frequency (or anti-resonance frequency) of the crystal in the lattice arm. Let f'_0 and F'_0 be the corresponding frequencies of the crystal in the series arm. Then in case (1) F_1, F_2 and F_0 (or f_0) are all different; in case (2) $F_1=F_2$ but F_0 (or f_0) is different; in case (3) $F_1=f_0$ or F_0 but F_2 is different; and in case (4) $F_1=F_2=F_0$ (or f_0).

2. NATURE OF REACTANCES IN SERIES AND LATTICE ARMS

Fig. 5 shows the reactance-frequency characteristics of the series and lattice arms drawn for the four cases mentioned above. The curve marked I refers to the characteristic of the negative impedance element together with crystal (mounted between electrodes) in parallel in the series arm; the curve marked II_1 refers to that of the crystal (mounted between electrodes) in the lattice arm and the curve marked II_2 refers to that of the negative impedance element in the lattice arm.

The nature of the reactance characteristic of the negative impedance element together with crystal (mounted between electrodes) in parallel in the series arm can be shown to follow from the consideration given below.

The impedance of the crystal (mounted between electrodes) in the series arm is given by

$$ {}_cZ'_1 = \frac{-j/\omega C'_0(1-\omega^2 L'_0 C'_0)}{1-\omega^2 L'_0 C' + C'/C'_0}. \tag{21}$$

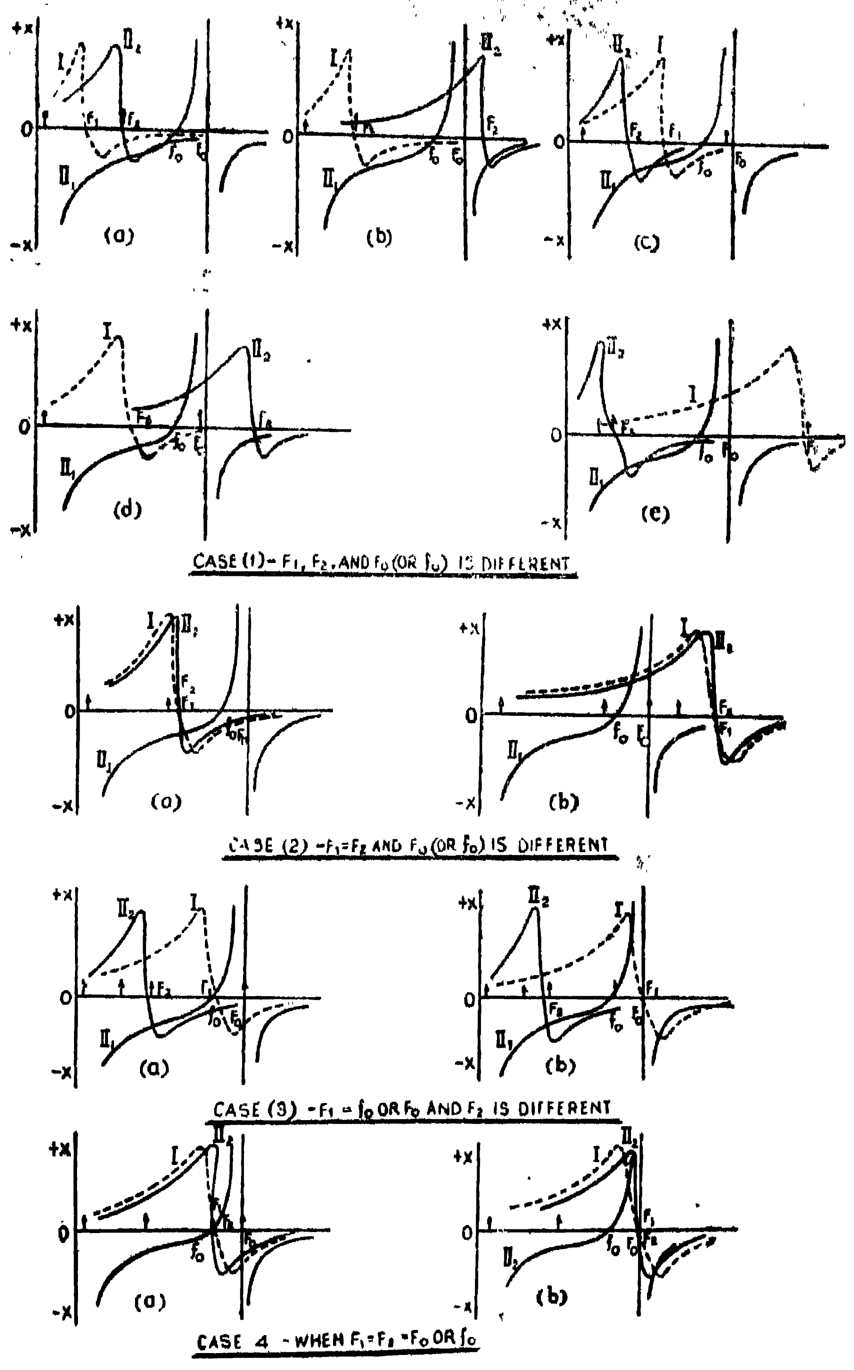


FIG. 5

Substituting C'_0k for C' in (21) and neglecting '1' in comparison to 'k' in the denominator, we have

$$ {}_0Z'_1 \simeq -\frac{j}{\omega C'_0 k}\,. \qquad \ldots \quad (22)$$

Hence the impedance of the crystal (mounted between electrodes) may be approximately regarded as purely reactive due to a capacitance C'_0k, and the usual equivalent circuit of the crystal (mounted between electrodes) may be replaced by the effective capacitance C'_0k. Since this capacitance is shunting the capacitance C_1 of the $L_1 - C_1$ parallel resonant circuit of the negative impedance element in the series arm, the total capacitance in parallel resonant circuit is now $C_1 + C'_0k$. Hence the reactance component of the negative impedance element shunted by the crystal in the series arm can be written as follows similar to that in equation (3).

$$_{N}X'_1 = \frac{L_1(C_1+C'_0k)\left|\omega L_1 - \dfrac{1}{\omega(C_1+C'_0k)}\right|}{(C_1+C'_0k)^2\left[\omega L_1 - \dfrac{1}{\omega(C_1+C'_0k)}\right]^2 + \dfrac{L_1^2}{m}\omega^4} \quad \ldots \quad (23)$$

When $\omega < \frac{1}{\sqrt{L_1(C_1+C'_0k)}}$, $\omega L_1 - \frac{1}{\omega(C_1+C'_0k)}$ will be negative and hence $_{N}X'_1$ will be positive; when $\omega = \frac{1}{\sqrt{L_1(C_1+C'_0k)}} = 2\pi F_1$, $_{N}X'_1 = 0$; and when $\omega > \frac{1}{\sqrt{L_1(C_1+C'_0k)}}$, $\omega L_1 - \frac{1}{\omega(C_1+C'_0k)}$ will be positive and hence $_{N}X'_1$ will be negative. As frequency is increased from a point before F_1, the magnitude of the positive reactance has been found to increase at first till a frequency $F_1 - \phi'$ and then to decrease rapidly to zero value at F_1. After the resonance frequency is passed the reactance is of negative sign and its magnitude at first increases till a certain frequency $(F_1 + \phi'')$ and then decreases to low value as the frequency is further increased. It will be noted that the nature of variation of the reactance of the negative impedance element shunted by the crystal (mounted between electrodes) in the series arm is *similar* to that of the negative impedance element in the lattice arm.

In all cases and sub-cases discussed below the cut-off frequencies indicated are for conditions in which the effects of resistance components of the impedances in series and lattice arms have been neglected. If the impedances are taken to consist of both resistance and reactance components, the effective cut-off frequencies will be different.

Take case (1) in which F_1, F_2 and F_0 (or f_0) are all different. (*a*) shows the condition when $F_1 < F_2 < f_0$ (or F_0), and F_1 and F_2 are nearer to each other. The probable positions of cut-off frequencies are indicated by arrows. It will be seen that at F_1 Mc/s the characteristic impedance will be zero unless the resistance component of the impedance in the series arm has an appreciable value. The condition can give a very wide band with slight reflection effect about F_1.

(*b*) shows the condition when $F_1 < f_0$ (or F_0) $< F_2$, and F_1 and f_0 and also F_2 and F_0 are nearer to each other. The probable positions of cut-off frequencies are indicated by arrows. This condition is expected to give a band-pass filter of lesser band-width than that of (*a*).

(*c*) shows the condition when $F_2 < F_1 < f_0$ or F_0. If F_1 and f_0 are nearer to each other, the band-width is expected to be very large.

(*d*) shows the condition when $F_1 < f_0$ (or F_0) $< F_2$, and F_2 is farther from F_0. This condition will give a very wide band band-pass filter of cut-off frequencies marked by arrows.

(*e*) shows the condition when $F_2 < f_0$(or F_0)$< F_1$, and F_1 is farther from F_0. This condition will give a very wide band band-pass filter of cut-off frequencies marked by arrows. There may be slight waviness in the total insertion loss characteristic between f_0 and F_0.

Case (2) shows the condition when $F_1 = F_2$, and F_0 (or f_0) is different. The band-widths are marked by arrows when F_1 (or F_2) is less than f_0 and also when F_1 (or F_2) is greater than F_0. The band-width in the latter case may be less than that of the former case if there be a large attenuation in the total insertion loss characteristic about F_0.

Case (3) shows the condition when $F_1 = f_0$, and F_2 is different as well as when $F_1 = F_0$, and F_2 is different. Two band-pass filters of different transmission band widths can be obtained under this condition and they could be made to give a single band-pass filter of larger band-width by altering the condition slightly in certain ways.

Case (4) shows the condition when $F_1 = F_2 = f_0$ and also when $F_1 = F_2 = F_0$. It will be seen that in the former case there may be two band-pass filters one of large and another of small transmission band width whereas in the latter case there may be only one band-pass filter. The probable cut-off frequencies of the band-pass filters are indicated by arrows.

Among the subcases of case (1) the sharpness of cut-off is expected to be more or less the same on both the sides in (*c*), (*d*) and (*e*) and different on both the sides in (*a*) and (*b*). For case (2), it is expected to be nearly the same in (*b*) and different in (*a*) on both the sides. For case (3) it is expected to be different for the band-pass filter involving the lower frequency range and same for the one involving the higher frequency range on both the sides. For case (4) it is expected to be different for the band-pass filter involving the lower frequency range and same for the one involving the higher frequency range on both the sides in (*a*), and it is expected to be different on both the sides in (*b*).

3. EQUIVALENT T-SECTION OF THE ORIGINAL LATTICE SECTION

A general section in which the resonance frequency of the equivalent parallel resonant circuit in the stabilised negative impedance element in the series arm, the frequency of the crystal in the lattice arm and the resonance frequency of the parallel resonant circuit in the stabilised negative impedance element in the lattice arm are all different is shown in Fig. 4(*a*) and its effective equivalent is shown in Fig. 4(b). Neglect the resistances in various inductances in series and lattice arms (that is, neglect R'_0, R_1, R_0 and R_2).

If Z_A be the total impedance in *each* series arm and Z_B the total impedance in *each* lattice arm, then we have

$$Z_A = \frac{j\omega L_1 R'_a}{R'_a[1-\omega^2 L_1(C_1+C'_0 k)] - j\omega L_1} \qquad \dots \quad (24)$$

$$Z_B = -\frac{j}{\omega C_0 k} + \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2}. \qquad \dots \quad (25)$$

If Z_1 and Z_2 be the total series and total shunt impedances respectively of the equivalent T-section in Fig. 4(c), then

$$Z_1 = \frac{2j\omega L_1 R'_a}{R'_a[1-\omega^2 L_1(C_1+C'_0 k)] - j\omega L_1} \qquad \dots \quad (26)$$

$$Z_2 = \tfrac{1}{2}\left[-\frac{j}{\omega C_0 k} + \frac{j\omega L_2 R_a}{R_a(1-\omega^2 L_2 C_2) - j\omega L_2} - \frac{j\omega L_1 R'_a}{R'_a[1-\omega^2 L_1(C_1+C'_0 k)] - j\omega L_1}\right]. \qquad \dots \quad (27)$$

4. CHARACTERISTIC IMPEDANCE

Separating the real and imaginary portions in the expressions on the R.H.S. of equations (26) and (27) and putting

$$a = \frac{\omega^2 L_1^2 R'_a}{R'^2_a[1-\omega^2 L_1(C_1+C'_0 k)]^2 + \omega^2 L_1^2} \qquad \dots \quad (28)$$

$$b = \frac{\omega L_1 R'^2_a[1-\omega^2 L_1(C_1+C'_0 k)]}{R'^2_a[1-\omega^2 L_1(C_1+C'_0 k)]^2 + \omega^2 L_1^2} \qquad \dots \quad (29)$$

$$a' = \frac{\omega^2 L_2^2 R_a}{R_a^2[1-\omega^2 L_2 C_2]^2 + \omega^2 L_2^2} \qquad \dots \quad (30)$$

and

$$b' = \frac{\omega L_2 R_a^2[1-\omega^2 L_2 C_2]}{R_a^2[1-\omega^2 L_2 C_2]^2 + \omega^2 L_2^2} \qquad \dots \quad (31)$$

we have

$$Z_1 = 2(a+jb) \qquad \dots \quad (32)$$

and

$$Z_2 = \tfrac{1}{2}\left[(a'-a) + j\left(b'-b-\frac{1}{\omega C_0 k}\right)\right]. \qquad \dots \quad (33)$$

Therefore finally the characteristic impedance of the equivalent T-section is given by

$$Z_0 = \sqrt{\left[aa' - bb' + \frac{b}{\omega C_0 k}\right] + j\left[a'b + ab' - \frac{a}{\omega C_0 k}\right]} \qquad \dots \quad (34)$$

5. NATURE OF THE CHARACTERISTIC IMPEDANCE

In order to estimate the relative values of real and imaginary terms under the radical sign in (34) over a frequency range, it is desirable to consider a typical ultra-wide band section of class II. Take a section in which $L_1=10\times10^{-6}$H, $C_1=19\times10^{-12}$F, $C'_0=.015\times10^{-12}$F, $k=140$, $|R'_a|$* $=2\times10^3$ ohms, $L_2=100\times10^{-6}$H, $C_2=.28\times10^{-6}$F, $C_0=.011\times10^{-12}$F, $|R_a|$† $=7.05\times10^3$ ohms, $F_2=30$ Kc/s, $F_1=11.5$ Mc/s, $f_0=7.17$ Mc/s and $F_0=7.24$ Mc/s.

Table IV shows the values of a, b, a', b' and Z_0 as well as of the real and imaginary terms under the radical sign in (34).

TABLE IV

Frequency in Mc/s	$\frac{1}{\omega C_0 k}$	a	b	a'	b'	$aa'-bb'+\frac{b}{\omega C_0 k}$	$a'b+ab'-\frac{a}{\omega C_0 k}$	Approximate $\lvert Z_0\rvert$ ohms
.05	22.6×10^5	0.005	3.15	43×10^{-3}	−16.0	68×10^5	-0.113×10^5	2607
.5	2.26×10^5	0.5	31.5	18×10^{-5}	−1.15	72.45×10^5	-1.16×10^5	2690
1	1.13×10^5	2	63	4.6×10^{-5}	−0.6	71.2×10^5	-2.26×10^5	2668
2	$.56\times10^5$	9	134	1.12×10^{-5}	0.27	80.4×10^5	-5.4×10^5	2840
3	$.377\times10^5$	21	202	0.5×10^{-5}	−0.19	76.8×10^5	-7.98×10^5	2780
4	$.28\times10^5$	43	292	0.29×10^{-5}	−0.14	81.8×10^5	-12.04×10^5	2910
5	$.226\times10^5$	77	383	0.19×10^{-5}	−0.12	88.1×10^5	-17.7×10^5	2983
6	$.188\times10^5$	144	529	0.125×10^{-5}	−0.095	100.5×10^5	-27.36×10^5	3240
7	$.16\times10^5$	240	627	0.09×10^{-5}	−0.08	100.3×10^5	-38.4×10^5	3286
8	$.14\times10^5$	483	871	0.073×10^{-5}	−0.071	122.4×10^5	-67.6×10^5	3741
10	$.113\times10^5$	1600	806	0.046×10^{-5}	−0.058	91.1×10^5	-180.8×10^5	4527

It will be seen from Table IV that the imaginary term under the radical sign in (34) varies from 0.16% to about 27% of the real term under the same sign over the range 50 Kc/s—6Mc/s and therefore Z_0 may be regarded more or less as pure resistance between those limits. At frequencies higher than 6 Mc/s, the imaginary term becomes greater and greater percentage of the real term and Z_0 will be an impedance with both resistance and reactance components, and further beyond a certain frequency Z_0 will be highly reactive giving conditions for the attenuation band. Further it will be noted that the magnitude of Z_0 varies in a wavy manner in the transmission band—at first increasing, then decreasing, then increasing,

* Magnitude of negative resistance in series arm at 5 Mc/s.

† Magnitude of negative resistance in lattice arm at 5 Mc/s. It will be noted that in actual case, $|R_a|$ for both series and lattice arm will vary irregularly with frequency over such an ultra-wide band (*i.e.*, 30 Kc/s–11.5 Mc/s) as under consideration at present. The values of $|R_a|$ taken from the mean curve (following the inverse square law) for this large range are liable to introduce great errors. Hence $|R_a|$ at 5 Mc/s has been taken for both cases in calculation.

again decreasing and finally increasing with frequency. At least for the frequency range over which Z_0 is more or less of non-reactive type, Z_1 and Z_2 can roughly be taken to be pure reactances of opposite signs. Hence over this range,

$$Z_1 \sim j.2b \qquad \ldots \quad (35)$$

$$Z_2 \sim -\frac{j}{2}\left[b+\frac{1}{\omega C_0 k}-b'\right]. \qquad \ldots \quad (36)$$

The characteristic impedance Z'_0 arrived at from (35) and (36) will be

$$Z'_0 = \sqrt{\left[\frac{b}{\omega C_0 k}-bb'\right]} \qquad \ldots \quad (37)$$

which can be compared to the value Z''_0 obtained by neglecting $a'b+ab'-\frac{a}{\omega C_0 k}$ in (34),

$$Z''_0 = \sqrt{\left[aa'-bb'+\frac{b}{\omega C_0 k}\right]}; \qquad \ldots \quad (38)$$

Z'_0 is nearly equal to Z''_0, since aa' is very small.

6. CUT-OFF FREQUENCIES

If $\frac{Z_1}{Z_2}=0$, then $b=0$, *i.e.*, $\frac{\omega L_1 R'^2_a[1-\omega^2 L_1(C_1+C'_0 k)]}{R'^2_a[1-\omega^2 L_1(C_1+C'_0 k)]^2+\omega^2 L_1^2}=0$

which means $\omega=0$ or $\omega=\frac{1}{\sqrt{L_1(C_1+C'_0 k)}}$.

The value $\omega=0$ is inadmissible. Hence one of the cut-off freqeuecies say, 'f_1', is given by

$$f_1=\frac{1}{2\pi\sqrt{L_1(C_1+C'_0 k)}}. \qquad \ldots \quad (39)$$

In the case of the ultra-wide band section considered above $f_1 = 10.93$ Mc/s.

If $\frac{Z_1}{Z_2}=-4$, then $j.2b=2j\left[b+\frac{1}{\omega C_0 k}-b'\right]$, or $\frac{1}{\omega C_0 k}=b'$,

or $$\omega^4(C_2+C_0 k)L_2^2 C_2 R_a^2-\omega^2(2C_2R_a^2+C_0kR_a^2-L_2)L_2+R_a^2=0 \qquad \ldots \quad (40)$$

on evaluating b' from (31). Solving the biquadratic and putting $\omega=2\pi f_2$, we have

$$f_2=\frac{1}{2\pi}\left[\frac{(2C_2R_a^2+C_0kR_a^2-L_2)L_2\pm\sqrt{(2C_2R_a^2+C_0kR_a^2-L_2)^2L_2^2-4R_a^2L_2^2C_2(C_2+C_0k)}}{2L_2^2C_2R_a^2(C_2+C_0k)}\right]^{\frac{1}{2}} \qquad \ldots \quad (41)$$

where 'f_2' is the real admissible value.

Taking $C_2=0.28\times10^{-6}$F, $L_2=100\times10^{-6}$H, $C_0=.011\times10^{-12}$F, $k=140$, $|R_a|=7.05\times10^3$ ohms from the data given for the ultra-wide band section,

$$f_2=\frac{1}{2\pi}\sqrt{353.5\times10^8}$$

$$=30 \text{ Kc/s}.$$

Hence the cut-off frequencies of the ultra-wide band section considered are roughly 30 Kc/s and 10.93 Mc/s. If $\frac{F_0+F'_0}{2}$ Mc/s be the arithmetic mean of the overall frequencies of the crystals in series and lattice arms, then the cut-off frequencies will be $\frac{F_0+F'_0}{2}-3B'$ and $\frac{F_0+F'_0}{2}+2B'$Mc/s where B′ varies from 2.12 to 2.22 Mc/s.

7. ATTENUATION AND PHASE CONSTANTS

The ratio of the current at the input (I_1) to the current at the output (I_2) when the T-section is terminated by Z_0 in the transmission band is given by

$$\frac{I_1}{I_2}=1+\frac{Z_1}{2Z_2}+\frac{Z_0}{Z_2}.$$

Taking Z_1, Z_2 and Z_0 from equations (35), (36) and (38) respectively we have

$$\frac{I_1}{I_2}=1-\frac{2b}{b+\frac{1}{\omega C_0 k}-b'}+j.\frac{2\sqrt{aa'-bb'+b/\omega C_0 k}}{b+\frac{1}{\omega C_0 k}-b'}$$

$$=\frac{\frac{1}{\omega C_0 k}-b-b'}{b+\frac{1}{\omega C_0 k}-b'}+j.\frac{2\sqrt{aa'-bb'+b/\omega C_0 k}}{b+\frac{1}{\omega C_0 k}-b'}. \qquad \ldots (42)$$

It will be seen from Table IV that $/b'/$ is very small except over a small portion of the transmission band near the lower cut-off frequency and aa' and bb' are also very small in comparison to $\frac{1}{\omega C_0 k}$.

Therefore

$$\frac{I_1}{I_2}\simeq\frac{\frac{1}{\omega C_0 k}-b}{b+\frac{1}{\omega C_0 k}}+j.\frac{2\sqrt{b/\omega C_0 k}}{b+\frac{1}{\omega C_0 k}}. \qquad \ldots (43)$$

If the propagation constant $P=\log_e(I_1/I_2)=\alpha+j\beta$, where α and β are attenuation and phase constants respectively, then

$$\alpha=\log_e\frac{\sqrt{\left(\frac{1}{\omega C_0 k}-b\right)^2+\frac{4b}{\omega C_0 k}}}{\sqrt{\left(b+\frac{1}{\omega C_0 k}\right)^2}}=\log_e\frac{\sqrt{\left(b+\frac{1}{\omega C_0 k}\right)^2}}{\sqrt{\left(b+\frac{1}{\omega C_0 k}\right)^2}}=\log_e 1=0 \quad ..(44)$$

$$\beta=\tan^{-1}\frac{4b}{\omega C_0 k}\Big/\left(\frac{1}{\omega C_0 k}-b\right)\simeq\tan^{-1}4b \qquad \ldots (45)$$

since

$$b<<\frac{1}{\omega C_0 k}.$$

Hence there is no attenuation in the transmission band of the section. Actual measurements of network attenuation have shown *low loss* in transmission band for sections of this class.

The phase-shift angle (in degrees) is almost the same at all frequencies in the transmission band, as shown in Table V.

TABLE V

f in Mc/s	.05	.5	1	2	3	4	5	6	7	8	10
tan β	12.5	126	252	536	808	1168	1532	2116	2508	3496	3224
β			>	>	near	near	near	near	near	near	near
(degrees)	85°30′	89°33′	89°48′	89°54′	90°	90°	90°	90°	90°	90°	90°

IV. ACKNOWLEDGMENT

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